Mathematics 102 Geometry & Trigonometry

Professor Peter M. Higgins

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The purpose of this module is to revise elementary geometry along with properties of the trigonometric functions and their inverses as these will arise continually throughout this degree course. There is a revisitation of the Euclidean theorems, particularly the *Circle theorems*. Some topics however are introduced using complex number and *matrix* techniques, which can be very effective and enlightening, giving us new ways of dealing with these familiar functions.

There are several particular topics that the student may not have met before, including the *Theorems of Pappus* for finding *volumes and areas of revolution*, the *Eulerian polyhedral formula* and *vector* methods in geometry, including the use of the *scalar* or *dot product* and the *vector* or *cross product*.

Problem Set 1 Geometry

1. The lengths of the two longer sides of a right-angled triangle are respectively one inch shorter and one inch longer than three times the length of the shortest side. Find the length of the hypotenuse.

2. For the triangle of Question 1, find the length of the perpendicular from the hypotenuse to the opposite vertex.

3. $\triangle ABC$ is such that the midpoint M of side AB is equidistant from all three vertices. The lengths of sides AC and BC are 5×10^6 kilometres and 1.2×10^7 kilometres respectively. Find the length of side AB.

4. Let P be the point $(3, 4)$ and let L be a tangent from the unit circle centred at the origin passing through P. Let Q be the point where L touches the circle. Find the length of the line segment PQ.

5. Find the distance from the point $(\frac{1}{2}, -\frac{2}{3})$ to the nearest point that lies on the unit circle of Question 4.

6. A cable is laid around the Earth's equator (assumed to be a circle). It is then decided that the entire cable has to be raised one metre above the surface. By how much will the length of the cable need to be extended? Answer the question again, this time replacing the Earth by Jupiter.

7. A quadrilateral $ABCD$ is inscribed in a circle with $\angle DAB = 110°$ and $\angle ABC = 40°$. Find the values of the other two angles.

8. Let AB be one side of a regular eleven sided polygon and let C be another vertex of that polygon. Find the value of the angle $\angle ACB$.

9. What is the angle (in radians) between two adjacent edges of a regular polygon with n sides?

10. Which three regular polygons *tessellate*? (This means that the plane can be tiled with identical tiles of these shapes.)

Problem Set 2 Geometry II

1. How many edges has a *regular dodecahedron*?

2. What regular solid arises from the joining of the centres of the adjacent faces of a cube?

3. What regular solid is formed by the joining of the centres of the adjacent faces of an *octahedron*?

4. The midpoints of consecutive sides of a quadrilateral are A, B, C, and D with the line through A and B having equation $4x+3y=7$ and the co-ordinates of C being $(-1,-1)$. Find the equation of the line through C and D.

5. Draw three circles, each of unit radius, with each one touching the other two. What is the area of the *intersterces*? (The shape trapped between the circles, which has three sides, each of which is an arc of one of the circles.)

6. What is the area of the largest circle that can sit inside an equilateral triangle of side length 2?

7. Two circles of unit radius are placed such that the centre of one lies on the circumference of the other. What is the value of their overlapping area?

8. A point $P(x, y)$ is first reflected in the x-axis, and then the result of that is reflected in the y-axis. What single geometric transformation has P undergone?

9. For a regular n-gon, what is the value of the *exterior angle* formed by the extension of one side with an adjacent side?

10. What is the sum of all the exterior angles of the n-gon of Question 9?

Problem Set 3 Trigonometry

1(a) What is the period of the function

$$y = -9\sin(4x - \frac{\pi}{3})?$$

(b) What is the period and the maximum value of the function

$$y = 3 + 3\sin 3x \cos 3x?$$

2. Show that

$$\arctan 1 + \arctan 2 + \arctan 3 = \pi.$$

3. Find all values of $\theta\,(0 \leq \theta \leq 2\pi)$ such that $\sin\theta + \cos\theta = \sqrt{2}$.

4. Find the solution in the second quadrant to the following equation, giving your answer in radians to two decimal places.

$$\sec^2 x + 5\tan x + 2 = 0.$$

5. Find all values of θ such that

$$\sec(\theta + \frac{\pi}{6}) = 2.$$

6. Without the use of calculator or tables find $\sin(\arccos(\frac{9}{41}))$.

7. The side BC, CA, AB of a triangle ABC are of lengths $x+y$, x, $x-y$ respectively. Express $\cos A$ in terms of x and y.

8. For the triangle of Question 7, simplify the expression:

$$\sin A - 2\sin B + \sin C.$$

9. A triangle has sides of lengths $a = 5.2$, $b = 3.7$, and $c = 7.1$ units. Find, to the nearest degree, the smallest of the triangle's angles.

10. Let $AB = 10$, $BC = 9$ and $\angle CAB = 60°$. Find the two possible values of AC.

Problem Set 4 Trigonometry II

1. Evaluate the following expression (without explicit integration of terms)

$$\frac{\int_{-t}^{t} \sin(x + \frac{\pi}{4})\, dx}{\int_{-t}^{t} \cos x \, dx}.$$

2. What is the period of the function $y = 1 - 2\tan(5x - \frac{\pi}{4})$?

3. Use the identity:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 75°$ in the form $a + \sqrt{b}$, where a and b are integers.

4. By using a suitable trigonometric identity, find the value of $\cos 15°$ as a radical expression of integers.

5. Simplify $\arcsin x + \arccos x$.

6. Express $\sin(\cos^{-1} x)$ algebraically.

7. Find the exact value of

$$\sin[\sin^{-1}(\frac{2}{3}) + \cos^{-1}(\frac{1}{3})].$$

8. $\cos 20° \cos 40° \cos 80°$ is a rational number. Which one?

9. What is the cosine of the angle α between the faces of a regular tetrahedron?

10. A police patrol spots a trawler, *The Goblin*, to the north-east steaming directly north at 10 knots. Suspecting *The Goblin* of smuggling contraband the patrol boat sets off at 25 knots on an interception course. What is the bearing of the velocity vector of the police boat? (Give your answer to the nearest degree east of north.)

Problem Set 5 Trigonometry III

Standard trigonometric identities Recall the Euler formula

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

By taking the real and imaginary parts of both sides of the equation $e^{(u+v)i} = e^{ui}e^{vi}$ show that:

1. $\cos(u+v) = \cos u\cos v - \sin u \sin v$ and $\sin(u+v) = \sin u\cos v + \cos u\sin v$.

Recall the matrix M_θ for rotating a point (x, y) (written as a column vector) through an angle θ about the origin:

$$M_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

2. Derive the identities of Question 1 from the equality $M_{u+v} = M_u M_v$.

3. Use the fact that cosine and sine are even and odd functions respectively, together with Question 1 to show that $\cos(u - v) = \cos u\cos v + \sin u\sin v$ and that $\sin(u - v) = \sin u\cos v - \cos u\sin v$.

4. Hence show that $\cos 2u = \cos^2 u - \sin^2 u = 1 - 2\sin^2 u = 2\cos^2 u - 1$ and that $\sin 2u = 2\sin u\cos u$.

5. Show that $2\cos u\cos v = \cos(u + v) + \cos(u - v)$ and also that $2\sin u\sin v = \cos(u - v) - \cos(u + v)$.

6. Use previous results to show that $\cos x + \cos y = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$ and $\cos x - \cos y = -2\sin(\frac{x+y}{2})\sin(\frac{x-y}{2})$.

7. Similarly show that $\sin x + \sin y = 2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})$ and also that $\sin x - \sin y = 2\sin(\frac{x-y}{2})\cos(\frac{x+y}{2})$.

8. Next show that

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u\tan v}, \ \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u\tan v}.$$

9. By using suitable idenitities show that

$$\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4} = \frac{\sqrt{2 - \sqrt{3}}}{2}.$$

10. Find $\int \sin 7x\cos 8x\, dx$.

Problem Set 6 Periodic functions

A function $f : \mathbb{R} \to \mathbb{R}$ has *period* p if p is the least positive number such that $f(x) = f(x + p)$ for all x in the domain of the function. For example, $f(x) = \sin x$ has period 2π.

1. Show that if f has period p then $f(x) = f(x \pm p)$ for all $x \in \mathbb{R}$.

2. Show that if f has period p then

$$g(x) = a + bf(cx + d)$$

$(a, b, c, d \in \mathbb{R},\ b, c \neq 0)$ has period $\frac{p}{|c|}$.

3. Suppose that f has period p. Show that if for some $q > 0$, $f(x + q) = f(x)\, \forall x \in \mathbb{R}$ then $q = np$ for some $n \in \mathbb{Z}^+$.

Find the periods of the following trigonometric functions.

4. $\cos x$ and $\tan x$.

5. $1 + 2\sec(3x - \pi)$.

6. $\sin^2 x$ and $\tan^2 x$.

7. $3\sin x + 4\cos x$.

8. $\sin 3x \cos 8x$.

9. Show that if $f(x)$ is a periodic differentiable function, then so is $f'(x)$.

10. Show that $\sin(x^2)$ is not periodic.

Problem Set 7 Geometry III

1. What is the volume of a regular tetrahedron of side length 1 unit?

2. For a given value of c ($|c| \geq 1$), find the values of m such that the line $y = mx + c$ is tangent to the unit circle centred at the origin.

3. What is the minimum length for a mirror so that a person of height h can see their full reflection?

4. How can you, with one straight cut, slice a rectangular cake into two (equal) halves in such a way that each half also has an equal share of the chocolate circle on the cake that does not necessarily lie in the middle?

5. A chord AB of a circle has length equal to the radius. Let C be another point on the circle. What are the two possible values of $\angle ACB$?

6. What is the radius of the largest circle that can sit inside the first quadrant of the unit circle?

7. Determine the length of a diagonal of a regular pentagon of side length one unit as a radical expression of positive integers.

8. Let A, B and C be three vertices of a cube with BA and BC each a diagonal of a face. What is the angle $\angle ABC$?

9. What is the area of the ellipse with equation

$$\frac{(x+1)^2}{9} + \frac{(y+1)^2}{49} = 1?$$

10. For any convex polyhedron, the number of vertices V, edges E, and faces F satisfies the formula of Euler $V - E + F = 2$. Verify this formula for the five *regular solids*: the tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron.

Problem Set 8 Vectors

1. Find, to the nearest degree, the angle between the vectors, $\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ and $-6\mathbf{i}+3\mathbf{j}-\mathbf{k}$.

2. Find the co-ordinates of the vertex D of the parallelogram $ABCD$ if A is at $(-1, 2)$, B is at $(3, 4)$, and C is at $(0, -6)$.

3. Let $\mathbf{u} = 2\mathbf{i} -\mathbf{j} +3\mathbf{k}$ and $\mathbf{a} = 4\mathbf{i} -\mathbf{j} +2\mathbf{k}$. Find the vector component of $\mathbf{u}$ along $\mathbf{a}$.

4. Find the component of $\mathbf{u}$ orthogonal (i.e. perpendicular) to $\mathbf{a}$, for the vectors of Question 3.

5. Find, to the nearest degree, the angle between the diagonal of a cube and one of its edges.

6. Find the equation of the line through the two points $A = (-1, -2, 2)$ and $B = (1, 7, 11)$, giving your answer in vector form $\mathbf{r} = \mathbf{a} + \mathbf{b}t$.

7. Answer the previous question, this time giving your answer in the form:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

8. Find the equation of the plane that contains the point $(-1, -2, 5)$ and is normal to the vector $\mathbf{n} = 5\mathbf{i} + 3\mathbf{j} - 8\mathbf{k}$.

9. Calculate $\mathbf{a} \times \mathbf{b}$, where $\mathbf{a} = (1, 7, 2)$ and $\mathbf{b} = (-1, 2, 5)$.

10. Find a unit vector perpendicular to both the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

Problem Set 9 Vectors II

1. Find the vector $\mathbf{x} = (x, y, z)$ such that $\mathbf{a} \cdot \mathbf{x} = -2$, $\mathbf{b} \cdot \mathbf{x} = 17$ and $\mathbf{c} \cdot \mathbf{x} = -7$, where $\mathbf{a} = (0, -1, 4)$, $\mathbf{b} = (1, 2, 3)$ and $\mathbf{c} = (-1, -1, 1)$.

2. *Direction cosines* Show that if $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is a vector, then $\frac{a}{|\mathbf{v}|} = \cos\alpha$, where α is the angle between $\mathbf{v}$ and the x-axis (and similarly for the other two axes). Moreover, the sum of the squares of these so-called direction cosines is 1.

3. Find a vector perpendicular to the plane of $P(1, -1, 0)$, $Q(2, 1, -1)$ and $R(-1, 1, 2)$ and give the equation of the plane in cartesian form.

4. Find the area of the triangle with vertices $A = (2, 2, 0)$, $B = (-1, 0, 2)$, and $C = (0, 4, 3)$.

5. Find the distance of the point $(2, 2, 2)$ to the plane $x - y + 4z = 9$.

6. Find the equation of the plane containing the three points $(2, 4, 1)$, $(-1, 0, 1)$ and $(8, 2, -5)$.

7. Find the equation of the plane that passes through the points $(1, 1, 1)$ and $(2, 0, 3)$ and which meets the plane $x + 2y - 3z = 0$ at right angles.

8. Verify the determinant formula:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}.$$

9. Verify the formula for the *scalar triple product*:

$$\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}.$$

10. Use the result in Question 9 to find the volume of the box *(parallelpiped)* the sides of which are determined by the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{c} = -\mathbf{i} + 4\mathbf{j} + \mathbf{k}$.

Problem Set 10 Geometry IV

1. Find the line through the point $(3, 4, 1)$ parallel to the line $x = 1 + 3t$, $y = -t$, $z = 1$, expressing it with the parameter t eliminated.

2. Find the minimum distance between the solution line of Question 1 and the line given by $x = -2t$, $y = 1 + t$ and $z = 3 + 2t$.

3. Find the parametric equation for the line of intersection of the planes $x + y + z = 0$ and $2x - y + 4z = 5$.

4. Eliminate the parameter in Question 3 to give the non-parametric equation of the line of the two planes.

5. What is the distance of the point $(2, 0, -3)$ to the line $\mathbf{r} = \mathbf{i} + (1 + 3t)\mathbf{j} - (3 - 4t)\mathbf{k}$?

First Theorem of Pappus (ca. 400 AD). The volume of revolution generated by rotating a plane area A around an axis outside of A equals the area of A multiplied by the distance travelled by it centre of mass (the point on which it could rest and be in balance).

6. Use Pappus to find the volume of a *torus* (doughnut shape) formed by rotating a circle of radius r around a line a distance $d \geq r$ from the centre of the circle.

7. Use Pappus in reverse to find the centre of mass of a solid semi-circle by considering the volume generated when the semi-circle is spun around its own diameter.

Second Theorem of Pappus The surface area of a volume of revolution equals the perimeter times the distance travelled by the centroid of the perimeter curve.

8. Find the surface area of a torus of circular radius r, the centre of which is a distance $d \geq r$ from its axis.

9. Use the theorem in reverse to find the centroid of a semi-circular wire of radius r by considering the corresponding sphere.

10. Similarly find the surface area of a cone of height h and circular radius r by considering the cone as the surface generated when a straight line from its apex to its base revolves around the axis of the cone.

Hints for Problems

Problem Set 1

1 & 2. Draw a decent diagram and for Question 2, exploit that the area of a triangle is half base times height.

3. You need the fact that the angle in a semicircle is a right-angle. Can you see why?

4. Here you use the fact that a tangent to a circle meets the radius at right angles.

6. Work with a symbol for the unknown radius and see what happens.

7. Opposite angles in a *concyclic quadrilateral* sum to $180°$.

8. The angle at the circumference standing on a chord of a circle is half that at the centre.

Problem Set 2

4. Remember that the midpoints of the sides of *any* quadrilateral form a parallelogram.

Problem Set 3

3. Write the right hand side in the form $r\cos(x - a)$ and then solve.

4. Use a standard trig identity to convert this to a quadratic equation; then remember where the 2nd quadrant is.

6. Draw a right triangle with sides of length 9 and 41 so that $\cos x = \frac{9}{41}$ for an acute angle x of the triangle. Then use Pythagoras and the definition of $\sin x$ to give the answer (you won't need a calculator).

7. Use the Cosine Rule.

8. And now the Sine Rule.

Problem Set 4

1. Expand the integrand of the numerator and use the fact that sine is an

odd function.

6. Put $\theta = \cos^{-1} x$ and draw a suitable triangle.

7. Expand and use Question 6.

8. Multiply the expression by $\sin 20°$ and then use a suitable trig identity repeatedly.

10. You do not know the separation of the vessels but let t be the time till interception and draw a suitable distance and direction diagram.

Problem Set 5

9. Use identity (3).

10. Apply identity (7).

Problem Set 6

7. Having identified the period p, you need to show that p is the least positive number that is a solution to the periodicity equation.

9. Need to go back to the definition of derivative here.

10. Make use of Question 9.

Problem Set 7

1. The volume of a pyramid is $\frac{1}{3}$ base times height. Also you can make use of the result of Question 9 on Sheet 2.

2. Use the fact that the equation describing the intersection of tangent and circle has exactly one root. Alternatively, the tangent meets the radius line at the contact point at right angles.

5. The angle standing on a chord at the centre of the circle is twice that at the circumference.

6. The common tangent to the circles is at right angles to their radii.

7. Look for similar triangles in the diagram and you will not need trigonometry.

9. Area of an ellipse is πab.

Problem Set 8

1. Remember your dot products: if x is the angle and $\mathbf{u}$ and $\mathbf{v}$ are the vectors involved then $\cos x = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|}$.

2. There is a little trap here: draw a good picture and make sure that you work with $ABCD$ and not $ADBC$, which is the temptation.

3 & 4. Here you are asked to find the projection of $\mathbf{u}$ in the direction of $\mathbf{a}$ (whose value is the dot product of $\mathbf{u}$ with the unit vector in the direction of $\mathbf{a}$) and Question 4 can then be solved by subtraction. This task of resolution of a vector into parallel and orthogonal components is basic to all of mechanics.

8. The entries in the normal vector correspond to the respective coefficients of x, y, and z in the equation of the plane.

9 & 10 Are question involving the *cross-product* of two vectors.

Problem Set 9

1. Solve the corresponding set of 3×3 linear equations.

3. Think in terms of cross-products.

4. One way to do it is to use the cross-product formula: $A = \frac{1}{2}|\mathbf{AB} \times \mathbf{AC}|$.

5. The required distance d is given by $|(\mathbf{r_0} - \mathbf{r_1}) \bullet \mathbf{n}|$ where $\mathbf{r_0}$ is the given point, $\mathbf{r_1}$ is *any* point in the plane and $\mathbf{n}$ is a unit normal vector the plane.

6. Use a cross-product to find the coefficients of the variables for the equation of the plane.

10. The answer is, up to sign, the determinant of the array with rows $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

Problem Set 10

2. For two skew lines $\mathbf{x} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{y} = \mathbf{c} + s\mathbf{d}$ $(t, s \in \mathbb{R})$, their minimum separation is given by $d = |\mathbf{n}\bullet(\mathbf{c} - \mathbf{a})|$ where $\mathbf{n} = \frac{\mathbf{b}\times\mathbf{d}}{|\mathbf{b}\times\mathbf{d}|}$.

3. Take the cross-product of the normals to the planes.

5. Find a vector to the given point from a point on the line, then project that vector onto the direction of the line. Now write down a normal vector from the line through the given point and then find its length.

Answers to the Problems

Problem Set 1

1. 35. 2. $11 \cdot 35$. 3. $|AB| = 1.3 \times 10^7$km. 4. $2\sqrt{6}$. 5. $\frac{5}{6}$. 6. 2π metres. 7. $140°$. 8. $\frac{\pi}{11}$. 9. $\left(\frac{n-2}{n}\right)\pi$. 10. Triangle, square, and hexagon.

Problem Set 2

1. 30. 2. octahedron. 3. cube. 4. $4x + 3y + 7 = 0$. 5. $\sqrt{3} - \frac{\pi}{2}$. 6. $\frac{\pi}{3}$. 7. $\frac{4\pi - 3\sqrt{3}}{6}$. 8. $180°$ rotation about the origin. 9. $\frac{2\pi}{n}$. 10. 2π.

Problem Set 3

1. $\frac{\pi}{2}$. 2. $\frac{\pi}{3}, \frac{9}{2}$. 3. $\frac{\pi}{4}$. 4. 2.53 and 1.80. 5.$\theta = 2n\pi + \frac{\pi}{6}$, or $2n\pi - \frac{\pi}{2}$ $(n \in \mathbb{Z})$. 6. $\frac{40}{41}$. 7. $\frac{x-4y}{2(x-y)}$. 8. 0. 9. $30°$. 10. $5 \pm \sqrt{6}$.

Problem Set 4

1.$\frac{1}{\sqrt{2}}$. 2. $\frac{\pi}{5}$. 3. $2 + \sqrt{3}$. 4. $\frac{\sqrt{6}+\sqrt{2}}{4}$. 5. $\frac{\pi}{2}$. 6. $\sqrt{1 - x^2}$. 7. $\frac{2(1+\sqrt{10})}{9}$. 8. $\frac{1}{8}$. 9. $\frac{1}{3}$. 10. $29°$.

Problem Set 5

9. $\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$. 10. $\frac{1}{2}\cos x - \frac{1}{30}\cos 15x$.

Problem Set 6

4. 2π, 2π. 5. $\frac{2\pi}{3}$. 6. π 7. 2π. 8. $\frac{\pi}{3}$.

Problem Set 7

1. $\frac{\sqrt{2}}{12}$. 2. $\pm\sqrt{c^2-1}$. 3. $\frac{h}{2}$. 5. $30°$, $150°$. 6. $\sqrt{2}-1$. 7. $\frac{1+\sqrt{5}}{2}$. 8. $60°$. 9. 21π .

Problem Set 8

1. $121°$. 2. $(-4,-8)$. 3. $\frac{20}{7}\mathbf{i}-\frac{5}{7}\mathbf{j}+\frac{10}{7}\mathbf{k}$. 4. $-\frac{6}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}+\frac{11}{7}\mathbf{k}$. 5. $55°$. 6. $\mathbf{a}=-\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ and $\mathbf{b}=2\mathbf{i}+9\mathbf{j}+9\mathbf{k}$. 7. $\frac{x+1}{2}=\frac{y+2}{9}=\frac{z-2}{9}$. 8. $5x+3y-8z+51=0$.9. $(31,-7,9)$. 10. $\mathbf{u}=\frac{1}{2\sqrt{2}}(2\mathbf{i}-2\mathbf{k})=\frac{\sqrt{2}}{2}(\mathbf{i}-\mathbf{k})$.

Problem Set 9

1. $(2,6,1)$. 3. $6\mathbf{i}+6\mathbf{k}$. 4. $-10\mathbf{i}+5\mathbf{j}-10\mathbf{k}$. 5. $\frac{15}{2}$. 6. $4x-3y+5z=1$. 7. $-x+5y+3z=7$. 10. -10.

Problem Set 10

1. $\frac{x-3}{3}=4-y$, $z=1$. 2. $\frac{14}{9}$ 3. $\mathbf{r}=(5t,-1-2t,1-3t)$. 4. $\frac{x}{5}=\frac{y+1}{-2}=\frac{z-1}{-3}$. 5. $\frac{\sqrt{41}}{5}$. 6. $2\pi^2r^2d$. 7. $\frac{4r}{3\pi}$. 8. $4\pi^2rd$. 9. $\frac{2r}{\pi}$. 10. $\pi r\sqrt{r^2+h^2}$.