Persistence of recessive genes

The trait of eye colour (restricted here to brown or blue) is determined by parent genes, each of which is comprised of two alleles, the pairs being bb, bB, or BB, with b and B standing respectively for blue and brown. Each parent passes on one of their alleles to their child who will inherit blue eyes only if both parents pass on a b allele. For this reason the blue allele is called *recessive* with respect to the *dominant* brown.

Suppose that in Generation 0, the proportion of each of these types in the population is respectively r, s, and t, so that $r+s+t=1$. A bb parent is bound to pass on a b allele, and similarly a BB parent passes on a B, but a bB person has equal probablility of passing on a b as they have of passing on a B allele.

Are blue-eyed people doomed to die out? A little algebra shows not, and completely answers the following questions, the solutions to which could not be arrived at any other way. (We assume that, with respect to genotypes, parents match up randomly with their partners.)

Problem 1 What are the respective proportions of each genotype in the next generation, Generation 1?

Problem 2 Answer the same question for Generation 2, and for all subsequent generations.

Solution to persistence of recessive genes problems

Problem 1 The probability that a randomly chosen parent passes on a b allele is $p = r + \frac{s}{2}$, and so the proportion of children in Generation 1 of type bb will be p^2. Similarly the proportion of type BB will be q^2, where $q = t + \frac{s}{2}$.

Note that

$$p + q = (r + \frac{s}{2}) + (\frac{s}{2} + t) = r + s + t = 1$$

$$\Rightarrow (p + q)^2 = p^2 + q^2 + 2pq = 1.$$

It follows that the proportion of Generation 1 of type bB will be $1 - (p^2 + q^2) = 2pq$. In summary, the probabilities in Problem 1 can be expressed as:

$$(bb, BB, bB) = (p^2, q^2, 2pq), \text{ where } p = r + \frac{s}{2} \text{ and } q = t + \frac{s}{2}. \qquad (1)$$

Problem 2 For Generation 2, we apply the same reasoning but with r, s, and t respectively replaced by $p^2, 2pq$, and q^2. The proportion of bb children in Generation 2 will then be:

$$(p^2 + \frac{2pq}{2})^2 = (p^2 + pq)^2 = (p(p + q))^2 = p^2 \cdot 1^2 = p^2,$$

which is unchanged from that of Geneation 1. By the same reasoning, the proportion of BB offspring in the new generation will be q^2, and by subtraction that for the bB type will also remain at the same value as before, that being $2pq$.

Therefore we conclude that the proportions of each genotype for Generation 2, and indeed for every subsequent generation, remains the same as for Generation 1, as given in (1) .