Mathematics 105 Probability & Combinatorics

Professor Peter M. Higgins

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The opening sets here are a potpourri of problems featuring simple probability and counting questions, sometimes featuring *permutation* and *combination* principles. These include coin, card, and dice problems.

Set 5 is on *conditional probability* and *Bayes's theorem.* From Set 7 onwards we see the special distributions, both discrete and continuous, including the *binomial, hypergeometric, Poisson* and approximations of one distribution by another. The *continuous distributions* include the *exponential*, *uniform*, and *normal* distributions.

Sets 9 and 10 involve calculation of *moments* of these distributions and the introduction and application of the *moment generating function.* We finish with the *Markov* and *Chebyshev inequalities.*

Problem Set 1 Probability and combinatorics

1. Find the constant term in the expansion of $(y - \frac{1}{2y})^{10}$, giving your answer as a fraction.

2. The probability that a certain football player scores from the penalty spot is 0.8. What is the probability that he misses two penalties in succession?

3. The full time score of the football match between Nuffing and Plough Town was: Nuffing m - Plough n. How many different possible half time score-lines were there? (Here we regard a score of Nuffing 2 - Plough 1 as different from Nuffing 1 - Plough 2.)

4. Returning to the Nuffing versus Plough Town match , given that $m \geq n$, answer the question again, but this time regarding the scores as unordered pairs, that is to say a score such as $2 - 1$ counts just once no matter who were the winning team.

5. There are three leads to be plugged into three sockets with each lead having just one correct socket. If I plug in the leads at random, what is the probability that all three are connected wrongly?

6. How many squares are there on a chess board? (This includes not only the unit squares but the 2×2, 3×3 etc. squares also.)

7. How many *oblongs* are there on a chessboard? (That is, rectangles that are not square.)

8. Let $f(x) = kx^2(1 - x)$ for all $0 \leq x \leq 1$ and let $f(x)$ be identically equal to 0 elsewhere. Determine the constant k given that $\int_0^1 f(x)\,dx = 1$.

9. A knock-out tennis tournament begins with n players. Each round consists of pairing off the remaining players (if an odd number remain, one player is given a bye—that is to say he or she enjoys a free passage to the next round). Each player then plays the opponent with whom they have been paired. The losers retire from the tournament, while the winners progress to the next round until the champion is decided. How many matches are played during the tournament? [No lengthy calculation is needed to answer this.]

10. Continuing with Question 9, if two players are chosen at random, (their names are drawn from a hat) what is the probability that they play each other at some stage during the tournament?.

Problem Set 2 Probabilty and Combinatorics II

1, In terms of a single game of tennis, what is the significance of the sequence of numbers, $4, 5, 6, 8, 10, 12, \cdots, 2n, \cdots$?

2. A red die is rolled followed by a green die. What is the probability that the green die beats the red?

3. The famous diarist Samuel Pepys asked the following question of Isaac Newton. One man throws six dice and needs to throw at least one ace (ace = 1) while another has twelve dice and needs to throw at least two aces. Which player has the advantage?

4. Two cards are drawn simultaneously from a normal pack of 52 playing cards. What is the probability that at least one is a diamond?

5. A coin rolls across a chessboard and settles randomly. Given that the diameter of the coin equals the length of the sides of each of the squares, find the probability that the coin covers a corner of some square.

6. In a game of Russian roulette, what is the probability that the first player wins? (To explain, two players take turns to discharge a loaded revolver pointed at their own head. On each turn, there is one chance in six of the gun firing, and a player 'wins' by killing himself.)

7. Find the coefficient of x^5y^8 in the expansion of $(2x - y^2)^9$.

8. Express in a closed form

$$\sum_{k=0}^{n} k \binom{n}{k}.$$

9. A player pays £2 to the bank, rolls a pair of dice and wins the *difference* between the two numbers showing. What are the expected winnings of the bank?

10. The game in Question 9 is modified so that if the player throws double six, they win a free throw. Is this modified game fair?

Problem Set 3 Probability and Combinatorics III

1. When two ordinary dice are rolled, what is the probability of tossing doubles or a score of eight?

2. Three ordinary dice are rolled. What is the probability that two of the dice (and not all three) show the same?

3. The 'Ashes' cricket series between England and Australia consists of 5 matches. What is the probability that one or other of the captains wins the toss (which occurs at the beginning of each match) on at least 4 occasions?

4. Find a binary string of length 8 such that, when written in a circle, every possible substring of length 3 appears exactly once.

5. Show that a clockface may be partitioned into three parts by two straight lines in such a way that the sum of the numbers in each part is the same.

6. How many distinct arrangements are there of the letters of the word GENEVIEVE?

7. In how many ways can 12 people split up to form 2 teams of 6 for a volleyball game?

8. How many 4-member committees may be formed from 4 men and 6 women, where at least 2 must be women and Mr and Mrs Higgins cannot both serve on the committee?

9. How many 8-digit binary sequences are there with 6 1's and 2 0's?

10. A fair coin is tossed repeatedly and you win £20 if the sequence TTH appears before the sequence HTT, otherwise you lose £10. What is your expected winnings in this game?

Problem Set 4 Probability and Combinatorics IV

1. How many ways are there to sit 5 (different) boys and 5 (different) girls in a row?

2. Answer Question 1 with the row replaced by a round table.

3. Answer Question 2 again with the girls and boys alternating.

4. Answer Question 1 again but this time with indistinguishable boys and indistinguishable girls, which is to say how many ways can you arrange 5 B's and 5 G's in a row?

5. What is the probability that I have to toss a coin n times until a head appears?

6. What is the average number of coin tosses required before I see a head?

7. Repeat Question 5 but replace 1 head by 2 (not necessarily successive) heads.

8. Repeat Question 6 but replace 1 head by 2 (not necessarily successive) heads.

9. How many solutions are there in non-negative integers to the equation

$$x_1 + x_2 + x_3 + x_4 = 12?$$

10. Repeat Question 9 but with positive integer solutions only.

Problem Set 5 Conditional probability

Recall that the probability of an event A given that an event B has occurred, denoted $P(A|B)$ is defined as $\frac{P(A\cap B)}{P(B)}$.

1. Derive an expression (*Bayes' Rule)* for $P(A|B)$ in terms of $P(B|A)$, $P(A)$ and $P(B)$.

2. Two dice are rolled. What is the probability that the sum of the faces exceeds 8 given that at least one die showed 6?

3. A canvasser is told that 60% of the people in her ward vote for her party. She is also told that 50% of the supporters of her party own bicycles while the overall figure for cycle ownership for the rest is just 25%. What proportion of people in her ward own bicycles?

4. Continuing under the conditions of the previous question, if she knocks at a house of a person that she sees owns a bike, what is the probability that she is contacting one of her supporters?

5. It is known that 45% of women and 35% of men say they voted for the Extreme Centre Party. A voter chosen at random says they voted ECP. What is the probability that this voter is a woman? (Assume equal numbers of men and women voters in the population.)

6. Rain falls one day in three and when it does, my barometer shows rain with a probability of $0\cdot 7$, otherwise it has a $0\cdot 1$ probability of showing rain. Find the probability that, on a random day, the barometer shows 'rain'.

7. Given that the barometer indicates rain, what is the probability that it is actually raining?

8. Three urns X_1, X_2 and X_3 contain white, red and black marbles in respective numbers: $X_1 : 1W, 3R, 2B$, $X_2 : 3W, 1R, 1B$, $X_3 : 3W, 3R, 3B$. Two marbles taken (without replacement) from a randomly selected urn are first white, then red. Find the probabililty that they came from X_2.

9. A medical test will produce a positive result if a patient has a certain disease and has a 5% probability of being positive otherwise. One person in one thousand suffers from this disease. A randomly chosen person tests positive. What is the probability that this person has the disease?

10. A bag contains two red and three blue marbles. You choose one marble at random and put it aside. Then you choose another and see that it is red. What is the probability that the first marble chosen was also red?

Problem Set 6 Probability and Combinatorics V

1. A particle P moves randomly along the x-axis either one step to the right or the left, and is equally likely to move either way. Given that P starts at the origin and there are absorbing barriers at -1 and 2, what is the probability that P is absorbed on the right hand barrier?

2. (GCSE, 2015) Charlotte has a bag on n sweets, 6 of which are orange and the rest yellow. She takes one, and then another at random and eats them. Given that the probability that she took 2 orange sweets is $\frac{1}{3}$, show that $n^2 - n - 90 = 0$ and hence find the number of sweets in Charlotte's bag.

3. Given that the top four Premiership football teams all make it to the last 16 for the FA Cup, what is the probability that each of these four teams is drawn with another in that round?

4. If two people toss a coin n times each, what is the probability that the number of heads they toss will be equal?

5. There are 20 identical coins on a table with 10 showing heads. You are blindfolded and gloved so that you cannot tell if a coin is a head or a tail. How do you split the coins up into two sets of 10 with each set having the same number of heads?

6. After a major earthquake, tremors occur at random for the next few days. Given that the probability of at least one tremor in the next hour is 0.64 what is the probability of at least one tremor in the next half hour?

7. Two players A and B pick a subset from a set S of size n at random, meaning that the probability of choosing each subset is the same. What is the probablity that B chooses a set that is contained in the set picked by A?

8. Repeat Question 7 but this time we ask for the probability that A and B are disjoint.

9. Repeat Question 7 asking for the probability that A and B together cover S.

10. Again as in Question 7, we now choose k sets at random and ask what is the probabilty that the set of chosen sets are pairwise disjoint.

Problem Set 7 Discrete distribution problems

1. It is known that 60% of the voting population support the Total Action Party. What is the probability that more than 5 voters in a random sample of 8 are TAP supporters?

2. A box contains a very large number of red and yellow roses in the ratio of $1:3$. How many flowers need to be randomly selected in order to ensure that the probability of at least one red bulb is greater than $0 \cdot 95$?

3. There are 3 black and 7 white balls in a bag. Find the probability that when 3 balls are take from the bag, they are all white when
(a) the balls are drawn one at a time and replaced on each occasion;
(b) the balls are chosen successively but without replacement.

4. The mean number of bacteria per raindrop is known to follow a Poisson distribution with mean $\lambda = 4$. What is the probability that a randomly examined drop has fewer than 3 bacteria?

5. A book of 500 pages has 750 misprints distributed randomly throughout. What is the probability that pages 427 and 428 will contain no misprints.

6. *Hypergeometric distribution* Suppose that a bag contains N marbles, n of which are blue and the rest red. If a group of m marbles is taken from the bag at random what is the probability that k of them are blue?

7. Sum the probabilities of Question 6 for all values from $k = 0$ to $k = m$ to verify that they do indeed sum to 1.

8. It is known that 80% of the children of an African village are suffering from a certain eye infection. If a doctor examines 12 of the children, what is the most likely number of cases she will detect?

Poisson approximation to a binomial distribution. A binomial distribution $X \sim \text{Bin}(n, p)$ may be approximated by a Poisson distribution with the same mean $\lambda = np$. This approximation is good when $n > 50$ and $p < 0 \cdot 1$.

9. A factory produces electronic chips in boxes of 500. The probability that a chip is defective is $p = 0 \cdot 002$. Find the probability that the box contains exactly 2 defective chips.

10. Find the probability that at least 2 double sixes are obtained when a pair of dice is tossed 90 times.

Problem Set 8 Continuous distribution problems

1. Consider a process in which events happen at random in time with mean frequency λ. The number of events per unit time interval is then a Poisson random variable $X \sim \text{Po}(\lambda)$. Let T be the continuous random variable the value of which is the length of time between successive events. Show that the cumulative distribution of T is $F(t) = 1 - e^{-\lambda t}$ $(t \geq 0)$ and hence find the probability density function of T.

2. Accidents occur randomly on a highway at the rate of 3 per day. Find the probability that after one accident has occurred, a full day goes by before there is another.

3. Let X be a continuous random variable defined over the real line with mean μ and standard deviation σ. Find the mean and standard deviation of $Z = \frac{X-\mu}{\sigma}$.

4. Given that $Z = \frac{X-\mu}{\sigma}$, express the *probability density function* $g(x)$ of X in terms of $f(x)$, the pdf of Z.

Normal random variables The standard normal random variable $Z \sim N(0,1)$ has mean 0 and standard deviation 1. Its *cumulative distribution function* is denoted by $\Phi(x) = P(X \leq x)$ so that, for example, $\Phi(1) = 0 \cdot 8413$ means $P(Z \leq 1) = 0 \cdot 8413$. The pdf of Z is $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ $(-\infty < x < \infty)$.

5. Find the pdf of $X \sim N(\mu, \sigma)$, (which means that $\frac{X-\mu}{\sigma} \sim N(0,1)$).

6. How is $\Phi(x)$ related to $\Phi(-x)$?

7. Let $X \sim N(50, 8)$. Find $P(|X - 50| < \sqrt{8})$.

8. It is known that 35% of the population have brown eyes. Use a normal approximation to the binomial to estimate the probability that in a sample of 400 people between 120 and 150 inclusive will have brown eyes.

9. A Geiger counter detects randomly decaying particles with a mean of $\lambda = 25$/second. Use a normal approximation to the underlying Poisson distribution to estimate the probabilty of between 23 and 27 (inclusive) detections in a particular second.

10. The result of a stress test X is known by long experience to have a standard deviation of $1 \cdot 3$ but the mean has been changed to a new value μ. It is required that a 95% confidence interval be found for $\mu$with total width less than 2. Find the least number of tests n required to ensure this.

Problem Set 9 Moments of random variables I

1. Let X be a random variable over the real line with mean μ and variance σ^2. Find the value of a that minimizes the expectation

$$\mathbb{E}((X-a)^2).$$

2. The *binomial random variable* $X \sim B(n,p)$ on the set $\{0,1,\cdots,n\}$ is the discrete random variable with probability function:

$$P(X=k) = \binom{n}{k} p^k q^{n-k}$$

where $0 \le p \le 1$ and $q = 1-p$. Find its mean, $\mathbb{E}(X)$.

3. Find the *first factorial moment* of $X = B(n,p)$ of Question 2, which is to say the value of $\mathbb{E}(X(X-1))$.

4. Use Question 2 and Question 3 to find the variance, σ^2 and standard deviation, σ of $X \sim B(n,p)$.

5. The *Poisson random variable* $X \sim P(\lambda)$ on the set of all non-negative integers $\{0,1,\cdots,k,...\}$ is the discrete random variable given by:

$$P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!},$$

where $\lambda > 0$ is fixed. Find the mean, $\mathbb{E}(X)$ of the Poisson distribution.

6. Find the first factorial moment of $X \sim P(\lambda)$ of Question 5, which is to say the value of $\mathbb{E}(X(X-1))$.

7. Use Question 5 and Question 6 to find the variance, σ^2 and the standard deviation, σ of $X = P(\lambda)$.

8. The *exponential random variable* $X(\lambda)$ with parameter $\lambda > 0$ is the continuous distribution with pdf given by

$$f(x) = \lambda e^{-\lambda x} \ (x \ge 0).$$

Verify that $f(x)$ is a probability density function and find the cumulative distribution $F(x) = P(X \le x)$.

9. Find $\mathbb{E}(X)$ for the *standard normal distribution* $X \sim N(0,1)$ has pdf

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \quad -\infty < x < \infty.$$

10. Find the standard deviation of $X \sim N(0,1)$.

Problem Set 10 Moments of random variables II

The *Moment generating function* (mgf) of a random variable X is define to be $M(X) = \mathbb{E}(e^{tX})$, $(t \in \mathbb{R})$ whenever this expectation exists. In consequence of the definition, $\mathbb{E}(X^k) = M^{(k)}(0)$, where $M^{(k)}$ denotes the kth derivative.

1. Find the mgf of the standard binomial distribution $B(n, p)$

2. Use the result of Question 1 to find the mean and variance of $B(n, p)$.

3. Repeat Question 1 for the *uniform distribution* on $[a, b]$, which is the random variable X with pdf $f(x)$ given by $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$ and $f(x) = 0$ outside this interval.

4. Find the mean of the uniform distribution by letting $t \to 0$ in $M'(t)$.

5. Find the mgf of the Poisson distribution $X(\lambda)$.

6. Use the mgf method to obtain the mean and variance of the Poisson distribution.

The *skewness* of a random variable measures how asymmetric is its distribution and is defined as

$$\kappa = \mathbb{E}\big(\frac{X - \mu}{\sigma}\big)^3.$$

7. Find the skewness of the Poisson random variable with parameter λ.

8. *Markov's Inequality:* Let X be a discrete random variable on the non-negative integers with mean μ. Show that

$$P(X \geq a) \leq \frac{\mu}{a}.$$

9. *Chebyshev's Inequality:* Let X be a discrete random variable with mean μ and non-zero variance σ^2. Use Markov's inequality to show that

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

10. Apply Chebyshev's Inequality to obtain an upper bound for the probability that the random variable X that represents the number of heads in 200 tosses of a fair coin takes on a value outside the range $100 \pm 10\sqrt{2}$.

Hints for Problems

Problem Set 1

1. Use the binomial theorem but you don't need to write out the entire expression, just go for the term involving y^0.

3. Remember that scores start at 0.

4. Just be careful not to double count!

5. Easiest to list all the possibilities and count the ones where all sockets are wrong.

7. A rectangle on a chessboard is formed by choosing two of the nine (yes nine, not eight) horizontal lines and two of the nine vertical lines. How many ways can you do this? From this number subtract the number of squares to get your oblongs.

10. Treat the *pairs* as the underlying set from which you are drawing at random and the problem is then quite simple.

Problem Set 2

1. Count the points.

2. Use symmetry.

5. Think in terms of where the centre of the coin falls.

6. Try to exploit the near symmetry in the problem.

Problem Set 3

6. Count the total number of permutations but remember that swapping identical letters does not change anything.

7. Choose the teams in order, but then remember that the two teams have no particular order.

8. First solve the problem without the restraint represented by the illegal couple, and then subtract the number of committees that violate the constraint.

10. Look at what is possible after the first two coin tosses.

Problem Set 4

6. & 8. The sums that arise can be found by substituting $x = \frac{1}{2}$ in derivatives of the geometric series $\sum_{n=0}^{\infty} x^n = (1-x)^{-1}$.

9. The number of non-negative integer solutions to $x_1 + x_2 + \cdots + x_n = m$ is $\binom{m+n-1}{m}$.

10. Substitute $x_i = y_i + 1$ and apply the formula of Question 9.

Problem Set 5

The answer to Question 1 is used throughout this problem set. Take time to name the events of interest with the symbols A, B etc and then use them.

2-6. It is important to introduce symbols for each event of interest and state the meaning of each one explicitly. Then apply the definition of conditional probability and the Bayes' formula as needed.

Problem Set 6

3. Begin by finding the number of ways to split a set of size $2n$ into pairs.

Problem Set 7

1. Identity the parameters n and p in for the Binomial distribution and apply that probability distribution.

2. A binomial distribution but you need to find the least value of the parameter n that will make the required condition true.

3. (a) is binomial but (b) is best seen as a ratio of binomial coefficients.

4. A Poisson distribution with $\lambda = 4$.

5. Treat as a single Poisson distribution.

7. Write the number of ways of choosing a group of m marbles from N as a sum of products of binomial coefficients corresponding to doing this with k blue marbles and $m - k$ red ones.

8. The mode of a binomial distribution $X \sim \text{Bin}(n, p)$ is generally $\lfloor (n+1)p \rfloor$ unless this is an integer in which case $(n+1)p - 1$ is also a modal value.

Problem Set 8

1. Let Y be the number of events in t time units. Then $Y \sim \text{Po}(\lambda t)$ and $P(Y = 0) = e^{-\lambda t}$.

3. Remember that in general, $E(h(X))$ means $\int_{-\infty}^{\infty} h(x) f(x)\,dx$ where $f(x)$ is the pdf of X.

8. Use $X \sim N(\mu, \sigma^2)$ where $\mu = np$ and $\sigma^2 = npq$. Use a continuity correction one your limits, meaning that for integers a and b estimate $P(a \leq X \leq b)$ by $P(a - \frac{1}{2} \leq X \leq b + \frac{1}{2})$. You will need that $\Phi(1 \cdot 101) = 0 \cdot 8465$ and $\Phi(2 \cdot 149) = 0 \cdot 9842$.

9. Use $X \sim N(\lambda, \lambda)$, again making use of a continuity correction in order to reduce rounding error. You will need $\Phi(0 \cdot 5) = 0 \cdot 6915$.

10. The 95% confidence interval is given by $\overline{x} \pm 1 \cdot 96 \frac{\sigma}{\sqrt{n}}$.

Problem Set 9

In these problems you should know the answer in advance. Use this to your advantage by setting up the sums, take out the known answer as a factor and use the Binomial theorem to simplify the remaining sum. This may require re-indexing of those sums to fit the statement of the Binomial theorem.

5. You need to use the exponential series: $\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{\lambda}$.

Problem Set 10

4. Cannot substitute $t = 0$ directly in $M'(t)$ so take the limit as $t \to 0$ and use L'Hopital's rule.

7. Work with the *third* factorial moment.

9. Apply Markov inequality to the random variable $(X - \mu)^2$.

Answers to the Problems

Problem Set 1

1. $-\frac{63}{8}$. 2. $0 \cdot 04$. 3. $(m+1)(n+1)$. 4. $\frac{1}{2}(n+1)(2m-n+2)$. 5. $\frac{1}{3}$. 6. 204. 7. 1092. 8. 12. 9. $n-1$. 10 $\frac{2}{n}$.

Problem Set 2

2. $\frac{5}{12}$. 3. $0 \cdot 619$. 4. $\frac{15}{34}$. 5. $\frac{\pi}{4}$. 6. $\frac{6}{11}$. 7. 4,032. 8. $n2^{n-1}$. 9. Bank's expected gain per roll is £0·055 . 10. Yes, it is now fair.

Problem Set 3

1. $\frac{5}{18}$. 2. $\frac{5}{12}$. 3. $\frac{3}{8}$. 4. 11100010. 5. The two lines give the partition $\{11, 12, 1, 2\}$, $\{5, 6, 7, 8\}$ and $\{3, 4, 9, 10\}$. 6. 7560 . 7. 462 . 8. 139 . 9. 28 . 10. $-$£2·50 .

Problem Set 4

1. 3,628,800. 2. 36,280. 3. 2,880. 4. 252. 5. $\frac{1}{2^n}$. 6. 2. 7. $\frac{n-1}{2^n}$. 8. 4. 9. 455. 10. 165.

Problem Set 5

1. $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$. 2. $\frac{7}{11}$. 3. $0 \cdot 25$. 4. $0 \cdot 75$. 5. 56.25%. 6. $\frac{3}{10}$. 7. $\frac{7}{9}$. 8. $\frac{2}{5}$. 9. $0 \cdot 0196$. 10. $\frac{1}{4}$.

Problem Set 6

1. $\frac{1}{3}$. 2. 10. 3. $\frac{1}{65}$. 4. $\frac{\binom{2n}{n}}{2^{2n}}$. 6. $0 \cdot 4$. 7. 8. & 9. $\left(\frac{3}{4}\right)^n$. 10. $\left(\frac{1+k}{2^k}\right)^n$.

Problem Set 7

1.$0 \cdot 315$. 2. 11. 3(a) $0 \cdot 343$ (b) $0 \cdot 292$. 4. $0 \cdot 238$. 5. $0 \cdot 0497$. 6. $\frac{\binom{n}{k}\binom{N-n}{m-k}}{\binom{N}{m}}$. 8. 10. 9. $0 \cdot 184$. 10. $0 \cdot 713$.

Problem Set 8

1. $f(t) = \lambda e^{-\lambda t}$. 2. e^{-3}. 3. $E(Z) = 0$, $\mathrm{Var}(Z) = 1$. 4. $\frac{1}{\sigma} f(\frac{x-\mu}{\sigma})$. 5. $\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\cdot\left(\frac{x-\mu}{\sigma}\right)^2}$. 6.$\Phi(x) + \Phi(-x) = 1$. 7. $0 \cdot 6826$. 8. $0 \cdot 8307$. 9. $0 \cdot 383$. 10. 7.

Problem Set 9

2. np. 3. $n(n-1)p^2$ 4. $\sigma = \sqrt{npq}$. 5. λ. 6. λ^2. 7. $\sqrt{\lambda}$. 8. $1 - e^{-\lambda x}$. 9. 0. 10. 1.

Problem Set 10

1. $M(t) = (p(e^t M - 1) + 1)^n$. 2. $\mu = np$, $\sigma^2 = npq$. 3. $\frac{1}{t(b-a)}\left(e^{tb} - e^{ta}\right)$, $t \neq 0; \mathbb{E}(e^0) = 1$. 4. $\mu = \frac{b+a}{2}$ 5.$M(t)e^{\lambda(e^t-1)}$.
6. $\mu = \sigma^2 = \lambda$. 7. $\kappa = \frac{1}{\sqrt{\lambda}}$. 10. $\frac{1}{4}$.